Emittance Growth Due to Tevatron Flying Wires *

M. Syphers and Nathan Eddy

June 21, 2004

Abstract

During Tevatron injection, Flying Wires have been used to measure the transverse beam size after each transfer from the Main Injector in order to deduce the transverse emittances of the proton and antiproton beams. This amounts to $36 + 9 = 45$ flies of each of 3 wire systems, with an individual wire passing through each beam bunch twice during a single "fly." Below we estimate the emittance growth induced by the interaction of the wires with the particles during these measurements. Changes of emittance from Flying Wire measurements conducted during three recent stores are compared with the estimations.

Upon passing through material of length d_{eff}, a particle with momentum p and speed v_s will undergo multiple Coulomb scattering and be deflected through a total angle θ_0 which, for a distribution of such scatterings, will have an rms value of[1]

$$\theta_0 = \frac{19.2 \text{ MeV}}{pv_s} \sqrt{\frac{d_{eff}}{L_{rad}}} \ [1 + 0.038 \ln(d_{eff}/L_{rad})]^{1/2}$$

where L_{rad} is the radiation length of the material. When projected onto one transverse plane, the rms scattering angle is $\langle \Delta\theta^2 \rangle^{1/2} = \theta_{x,y}^{rms} = \theta_0/\sqrt{2}$ and, neglecting the logarithmic term,

$$\langle \Delta\theta^2 \rangle = \left(\frac{13.6 \text{ MeV}}{pv_s} \right)^2 \frac{d_{eff}}{L_{rad}} \ .$$

During a Flying Wire measurement, a particle which passes through the wire material at a location where the amplitude function has value β will scatter and have its betatron amplitude changed from a_0 to a according to

$$a^2 = a_0^2 + \beta^2 \Delta\theta^2 - 2a_0\beta\Delta\theta \cos\phi$$

as illustrated in Figure 1. Summing over all the particles and dividing by the total number of particles, N,

$$\langle a^2 \rangle = \langle a_0^2 \rangle + \frac{N_w}{N} \left[\beta^2 \langle \Delta\theta^2 \rangle - 2\langle a_0 \rangle \beta \langle \Delta\theta \rangle \langle \cos\phi \rangle \right]$$

---

*This note is based upon a talk presented at a Tevatron Department Meeting in August 2002 (Syphers). Interest in the subject has arisen once again, along with recent emittance growth measurements.
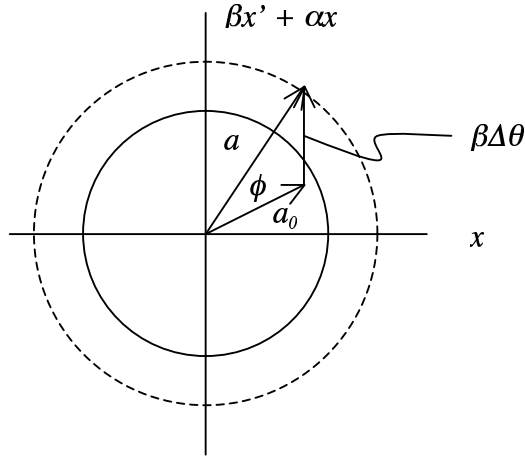
Figure 1: **Change of phase space amplitude at the wire location due to scattering.**

where N_w is the number of particles which interact with the wire. Here, N_w equals N times the probability of a particle passing through the wire during a measurement. The last term averages to zero. The resulting transverse rms beam size, σ, is given by $\sigma = \langle x^2 \rangle^{1/2} = \langle a^2/2 \rangle^{1/2}$ and thus the change in the variance of the transverse distribution is

$$\Delta\sigma^2 = \frac{1}{2}\frac{N_w}{N}\beta^2 \langle \Delta\theta^2 \rangle.$$

The Flying Wires are mounted on a fork which rotates about an axis of radius R and pass through the beam with a wire speed v as shown in Figure 2. Their transverse speed is thus,
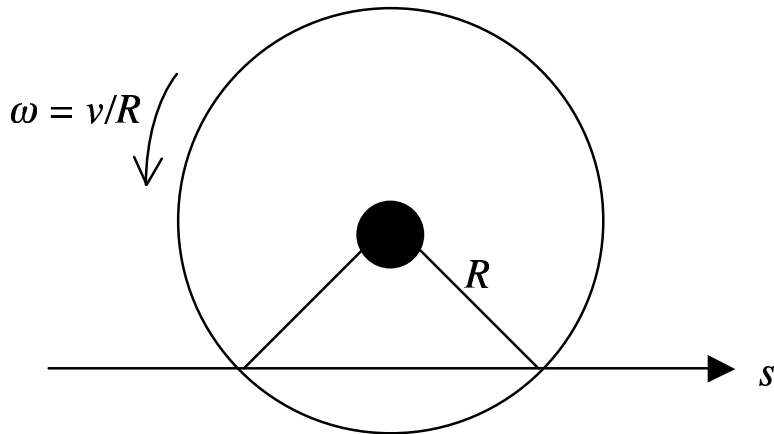

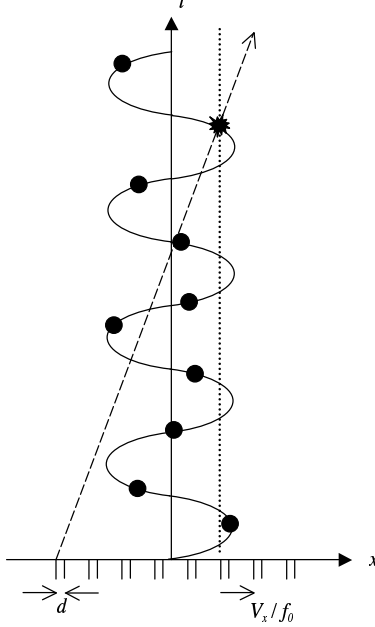Figure 2: **Schematic of Flying Wire. Beam passes to the right**

Figure 3: **Interaction of particle with a wire of thickness d. The particle is undergoing a betatron oscillation and appears on this plot as a "dot" each revolution, while the wire advances a distance v_x/f_0 each revolution. The open "star" indicates a "hit."**

approximately, $v_x = v/\sqrt{2}$. Assuming an arbitrary initial phase of a particle's betatron oscillation, as depicted in Figure 3, the probability of passing through the wire is then

$$N_w/N = \frac{d}{v_x} \cdot f_0$$

where f_0 is the revolution frequency.

A particle which passes through the wire will see an effective thickness d_{eff} which, when averaged over a circular wire cross section of diameter d, gives $d_{eff} = 2d/\pi$. So, putting everything together, the growth of the transverse 95% normalized emittance generated by a single Flying Wire which passes through the beam twice during one measurement is estimated to be

$$\Delta\epsilon_N = 2 \times \frac{6\pi \cdot \gamma(v_s/c)}{\beta} \times \frac{1}{2}\left(\frac{d}{v_x}\cdot f_0\right) \times \beta^2 \left(\frac{13.6 \text{ MeV}}{pv_s}\right)^2 \frac{(2d/\pi)}{L_{rad}}$$

or

$$\Delta\epsilon_N = \frac{12\sqrt{2}}{\pi}\beta\left(\frac{d^2}{L_{rad}}\right)\left(\frac{13.6 \text{ MeV}}{pv_s}\right)^2\left(\frac{f_0}{v}\right)\left(\gamma\frac{v_s}{c}\right)\pi$$

per fly, per wire.

The Tevatron uses Carbon wires, for which $L_{rad} = 18.8$ cm, and have diameter $d = 30$ μm. The locations of the wires have typical amplitude function values of $\beta \sim 80$ m, and the wires are "flown" at speeds of $v \sim 5$ m/sec. At the injection energy of 150 GeV, and with a revolution frequency of
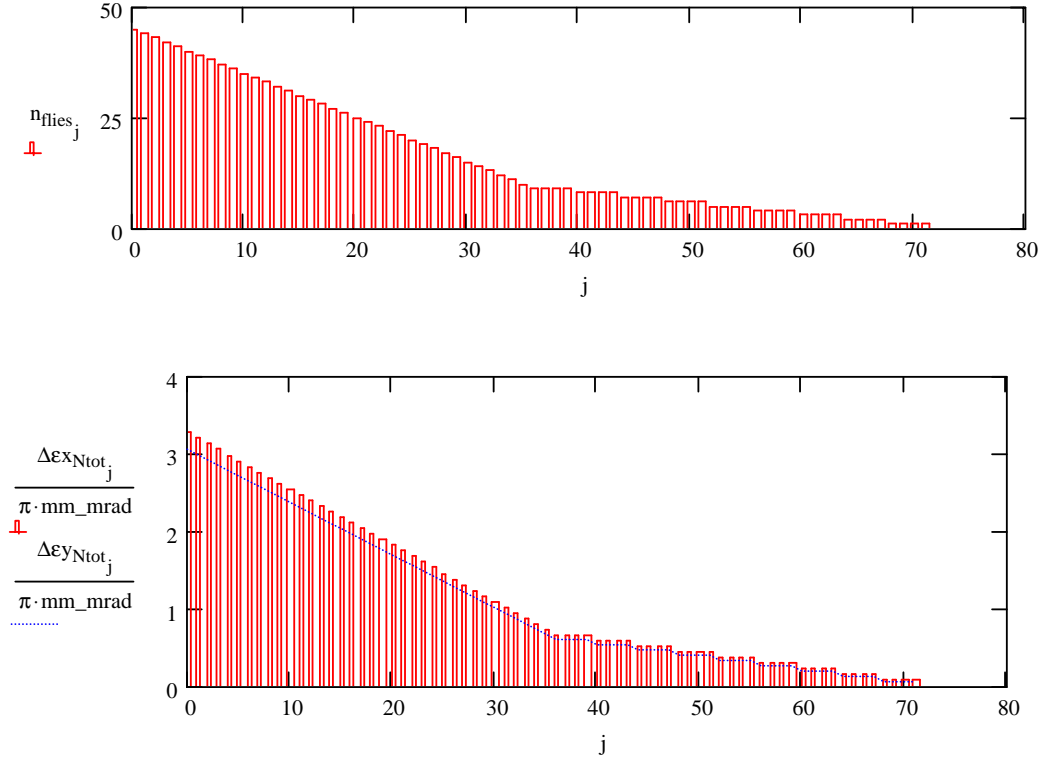
Figure 4: **Number of Flying Wire measurements versus bunch number (top) and resulting emittance growth (bottom) during typical Collider injection process. Bunches 0-35 are the proton bunches, numbers 36-71 are antiproton bunches.**

$(3 \times 10^8 \text{ m/sec})/(2\pi \text{ km}) = 47.7 \text{ kHz}$, the emittance growth would be

$$
\begin{aligned}
\Delta \epsilon_N &= \frac{12\sqrt{2}}{\pi}(80 \text{ m})\left(\frac{900 \times 10^{-6}\text{mm}^2}{188 \text{ mm}}\right)\left(\frac{13.6}{160 \times 10^3}\right)^2 \left(\frac{47.7 \times 10^3/\text{sec}}{5\text{m/sec}}\right)(160)\,\pi \\
&\approx 0.02 \; \pi \; \text{mm} - \text{mrad per fly, per wire,} \\
&\approx 0.06 \; \pi \; \text{mm} - \text{mrad per fly, (3 wires).}
\end{aligned}
$$

Note that the probability of a particle scattering through the wire during a measurement is $(d\sqrt{2}/v)f_0 \sim 40\%$.

For Collider operation in the Tevatron, 36 proton bunches are injected one by one and traditionally Flying Wire measurements are made after each injection. Nine transfers of four antiproton bunches are then injected with measurements taken after each transfer. So, the first proton bunch injected is subjected to 45 Flying Wire measurements, and the last four antiproton bunches experience only one fly at injection. The distribution of wire flies and subsequent emittance growth for the 72 bunches injected are shown in Figure 4. The mean emittance growth over the 72 bunches is 1.2π mm-mrad, with a maximum growth of 3.2π mm-mrad for Proton bunch No. 1. The average growth for the Proton bunches is 2π mm-mrad, while the average over the Antiroton bunches is 0.33π mm-mrad.
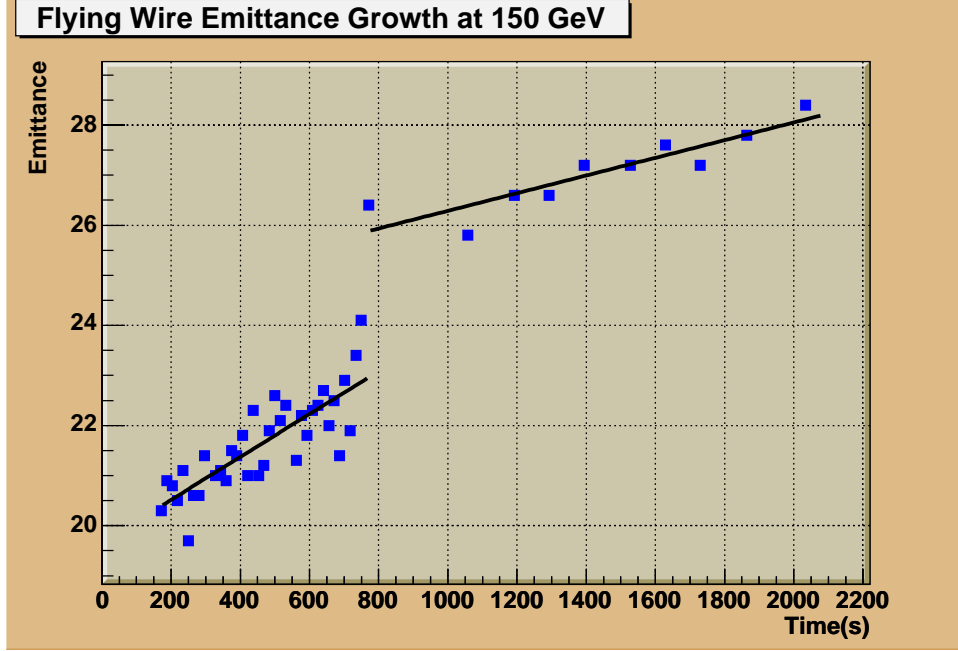
Figure 5: **Results of emittance measurements taken during Tevatron injection process. The data are for an individual representative bunch.**

Emittance measurement data taken during three Tevatron stores were used to determine the emittance growth per fly of the Flying Wire system. Both the vacuum and per fly emittance growth can be calculated from Tevatron flying wire data taken during the injection of protons and anti-protons into the Tevatron during a shot setup. The calculation is based on the assumption that

$$Emittance\ Growth = V\Delta t + F\ n$$

where $V \cdot \Delta t$ is the emittance growth due to the vacuum over a time interval Δt and $F \cdot n$ is the emittance growth from flying the wires n times. The analysis makes use of the fact that during shot setup, 36 protons are injected about 15 seconds apart then followed by 9 antiproton injections about 1 minute apart. By plotting the emittance as measured by the flying wires against time, one can clearly see two distinct slopes for proton injection and antiproton injection. An example plot and fit for a single bunch is shown in Figure 5. The time and number of flies in each case is known while a linear fit provides the emittance growth over the period in question. Plugging into the above equation yields a set of two equations with two unknowns which can be solved for the two emittance growth rates, V and F.

This technique can be applied to any proton bunch but loses sensitivity for successive bunches as they see fewer flys before antiproton injection. To estimate V and F to reasonable accuracy, this method is applied to the first proton bunches in 3 stores, numbered 3421, 3434, and 3436. Also data were collected from both the E11 horizontal and E11 vertical Flying Wires. While the E17 horizontal wire was also flown in each case it was not used for this study because it has much larger
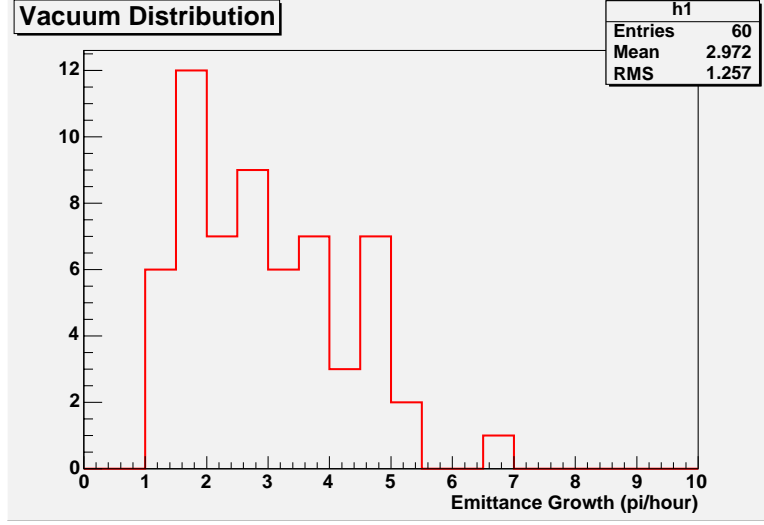
Figure 6: **Results of emittance measurements taken during Tevatron injection process.**

errors than the E11 wires. Figure 6 shows the resulting distribution for V, the emittance growth rate due to the vacuum. The spread is large but the average of 3.0π/hour is in good agreement with 2.9π/hour which has been previously calculated.[2] Figure 7 shows the resulting distribution for F, the emittance growth per fly. The average is $0.045 \pm 0.016\pi$/fly which is in reasonable agreement with the predictions presented above.
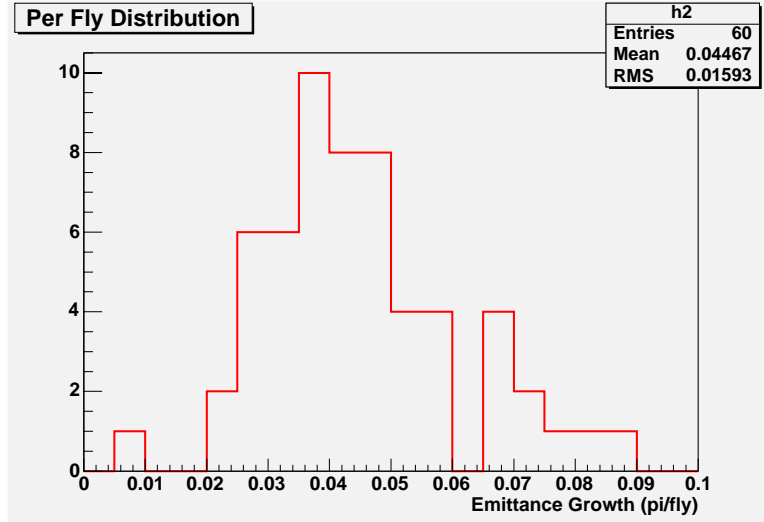

Figure 7: **Results of emittance measurements taken during Tevatron injection process.**

The authors would like to thank Stephen Pordes for his encouragement to have these notes written up.

References

[1] See, for example, "Review of Particle Physics, Part I," *Phys. Rev.* **D66**, 198-199 (2002).

[2] "Residual Gas, Emittance Growth and Beam Lifetime in the Tevatron at 150 GeV," V.A. Lebedev, L.Y. Nicolas, A.V. Tollestrup, Fermilab Report, Beams-doc-1155 (2004).